

Speaker: Kaj Nyström

Title: *Free boundary and inverse type problems for the p -Laplace Operator*

Abstract: In [C1,C2,C3] a theory for general two-phase free boundary problems for the Laplace operator was developed. In [C1] Lipschitz free boundaries were shown to be $C^{1,\gamma}$ -smooth for some $\gamma \in (0,1)$ and in [C2] it was shown that free boundaries which are well approximated by Lipschitz graphs are in fact Lipschitz. Finally, in [C3] the existence part of the theory was developed. In [LN1], [LN2] John Lewis and I study the corresponding problems for the p -Laplace operator and we generalize the results in [C1,C2] to the p -Laplace operator when $p \neq 2$, $1 < p < \infty$. Furthermore, in [LN3] we resolve a number of problems concerning regularity and free boundary regularity, below the continuous threshold, for the p -Laplace operator, $1 < p < \infty$, in domains $\Omega \subset \mathbf{R}^n$, $n \geq 2$, which are assumed to be Reifenberg flat and Ahlfors regular. In particular, in [LN3] we establish a p -harmonic version of the results established in [KT,KT1,KT2,KT3] concerning harmonic functions. We claim that all of these generalization are highly nontrivial due to the non-linear and degenerate character of the p -Laplace operator for $p \neq 2$. The purpose of this talk is to briefly discuss some of these recent results.

References

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- [LN1] J. Lewis and K. Nyström, *Regularity of Lipschitz Free Boundaries in Two-phase Problems for the p -Laplace Operator*, to appear in Advances in Mathematics.
- [LN2] J. Lewis and K. Nyström, *Regularity of Flat Free Boundaries in Two-phase Problems for the p -Laplace Operator*, submitted.
- [LN3] J. Lewis and K. Nyström, *Regularity and Free Boundary Regularity for the p -Laplace Operator in Reifenberg Flat and Ahlfors Regular Domains*, in preparation.