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SMS-BULLETINEN

Innehåll

Nästa Bulletin

Nästa Bulletin planeras komma ut i februari. Skicka gärna lokala nyheter till samfundets sekreterare Olof Svensson, secretary@swe-math-soc.se, senast 1 februari.

Resestipendier

SVeFUM – Stiftelsen för Vetenskaplig Forskning och Utbildning i Matematik - ledigförklarar härmed resestipendier för i Sverige bosatta matematiker av alla kategorier, dock lägst på doktorandnivå. Stipendier kan sökas för konferenser och andra resor med vetenskapligt syfte, ävensom för längre postdoc-vistelser i utlandet. Utdelade stipendier är personliga och utbetalas till stipendiatens privata konto. Ansökningar, ställda till SVeFUM, c/o Prof. Kjell-Ove Widman, sänds per e-post till `svefum(at)widman.ch` och bör innehållande en kort redogörelse för ändamålet med resan, budget, CV i kortform samt svenskt personnummer och kontonummer för utbetalning. Svenska examens- och anställningstitlar används i förekommande fall. För doktorander fordras rekommendationsbrev från handledare, skickat direkt till SVeFUM, liksom en lista över genomgångna kurser och ev. publikationer eller preprints. Sista ansökningsdag är 2021-02-28. Ev. frågor riktas till `svefum(at)widman.ch`

Stiftelsen utgår från att sökanden, genom att ansöka, godkänner att i ansökan angivna personuppgifter lagras i enlighet med stiftelsens principer, innebärande att radering sker efter tio år för sökande som tilldelats anslag, medan för ansökningar som helt avslagits gäller att uppgifterna normalt raderas senast året efter det verksamhetsår som ansökan gäller.

Lokala nyheter

Karlstads universitet

Nya doktorander

Markos Fisseha

Linköpings universitet, Linköping

Ny docent i optimeringslära

Nils-Hassan Quttineh

Ny biträdande universitetslektor i optimeringslära

Yurii Malitsky.

Lunds universitet

Nya doktorsavhandlingar

Kenneth Batstone, "Computer Vision without Vision — Methods and Applications of Radio and Audio Based SLAM", 2 oktober 2020.

Mälardalens högskola

Ny lektor

Achref Bachouch

Umeå Universitet:

Ny professor

Klas Markström

Nya lektorer

Anttii Perälä, Fredrik Ohlsson, Jonas Westin, Klara Stokes och Maryam Sharifzadeh

Nya doktorander

Ali Dadras, Hoomaan Maskan, Rebecka Andersson, Signe Lundqvist och Yujie Shen

Nya doktorsavhandlingar

Johan Strandberg, matematisk statistik, Icke-parametriska metoder för funktionella data, 30 oktober
Therese Kellgren, matematisk statistik, Hidden Patterns that Matter: Statistical Methods for Analysis of DNA and RNA Data, 16 oktober

Program, SMS års- och höstmöte, 20 november 2020

Mötet äger rum på zoom,
https://lu-se.zoom.us/j/62367688248?pwd=Qk5yckJ1U3NLNllpalp4NnBVb3hiQT09
Lösenord: SMS2020

14.00–14.45	John Andersson Presentation av Erik Lindgrens arbeten
15.00–15.45	Tobias Ekholm Presentation av Thomas Kraghs arbeten
15.50–16.20	David Lundberg
16.30	Årsmöte. Dagordning finns i separat dokument.

Dagordning för Svenska matematikersamfundets års- och höstmöte 2020

Mötet äger rum på zoom,
`https://lu-se.zoom.us/j/62367688248?pwd=Qk5yckJ1U3NLNllpalp4NnBVb3hiQT09`
Lösenord: SMS2020

1. Mötets öppnande
2. Val av mötesordförande och mötessekreterare
3. Val av två justeringspersoner
4. Fastställande av dagordningen
5. Framläggande av årsberättelse, balansräkning och revisionsberättelse
6. Frågan om beviljande av styrelsens ansvarsfrihet.
7. Val av två revisorer och revisorssuppleanter för verksamhetsåret 20/21
8. Val av tävlingskomitte för verksamhetsåret 20/21
9. Val av valberedning för verksamhetsåret 20/21
10. Plats för årsmötet 2021
11. Övriga frågor
12. Mötet avslutas

The

CAMS Thematic Program in Mathematical Physics, 2020–2021

Spectral Theory, Semi Classical Analysis and Condensed Matter Physics

which will be hosted by the Center for Advanced Mathematical Sciences (CAMS) at the American University of Beirut (AUB), between November 2020 and March 2021. This extended program, which includes a series of mini courses, seminars and an international conference, is organized by Wafaa Assaad (Laboratory of Mathematics, Doctoral School of Science and Technology (DSST), Lebanese University) and Ayman Kachmar (Department of Mathematics, Faculty of Science, Lebanese University), in close collaboration with CAMS and DSST. The program's conference is co-organized with M. Correggi (Politecnico di Milano). The program focuses on PDEs, spectral theory, semiclassical analysis and their applications in physics, in particular, in quantum mechanics, superconductivity, liquid crystals, and Bose–Einstein condensates.

DSST-LU students who attend the mini courses will be eligible to ECTS credits. Graduate students will be eligible to a certificate, issued by CAMS, in recognition of attending program courses.

Below is a brief summary of the program:

- Online mini courses (November 2020–March 2021):
  (1) A mini course by A. Kachmar (Lebanese University): *The principal eigenvalue of the magnetic Robin Laplacian in the disc* (November 3–10, 2020).
  Registration link
  (2) A mini course by W. Assaad (Lebanese University): *The principal eigenvalue of the Neumann Laplacian for uniform and discontinuous magnetic fields* (November 17–24, 2020).
  Registration link
  (3) A mini course by M. Wehbe (Lebanese University): *Harmonic morphisms between graphs* (December 1–8, 2020).
  Registration link
  (4) A mini course by N. Raymond (Université d'Angers): *Spectral measures and applications* (January 4–8, 2021).
  Registration link: TBA
  (5) A mini course by B. Helffer (Université de Nantes): *Semiclassical methods and tunneling effects* (January 17–28, 2021).
  Registration link: TBA
  (6) A mini course by M. Zerzeri (Université Sorbonne, Paris Nord): *Introduction to semi-classical microlocal analysis and spectral asymptotics* (March 1–12, 2021).
  Registration link: TBA
- Monthly seminars on mathematical physics. Further details regarding the seminars will be provided in due course.
- An international conference titled *Mathematics of Condensed Matter and Beyond* (MCMB, February 22–25, 2021), organized by W. Assaad, M. Correggi (Politecnico di Milano), and A. Kachmar.

Refer to the thematic program's webpage for further information and updates:
https://www.aub.edu.lb/cams/Pages/Semester.aspx
and the conference webpage: https://www.aub.edu.lb/cams/Pages/conferences.aspx

Registration for the mini courses is now open!

LOUISE PETRÉN — A MATHEMATICIAN WHOSE WORK WAITED FOR A CENTURY TO BE APPRECIATED

INNA S. EMELYANOVA

Abstract. Louise Petrén (Hedvig Louise Beata Petrén-Overton, 1880–1977) defended her doctoral thesis in mathematics in 1911 as the first woman in Sweden. It took nearly a hundred years for her thesis to be duly appreciated.

I was approached by Barbro Grevholm[1]. She sent her article "Louise Petrén, the first woman in Sweden to win a doctorate in mathematics" [1] with the request to write a review of the current assessment of Louise Petrén's work. The name was not unknown to me. In 2007 at the international conference "Modern Group Analysis" in Sweden I heard her grandson Lars Haikola[2] giving a talk on Louise Petrén's biography and personality. His paper was published in [2].

Louise Petrén (Hedvig Louise Beata Petrén[3]) was born on August 12, 1880 in the family of Edvard Petrén (Carl Daniel Edvard Petrén, 1825–1901), a church pastor in the Swedish parish of Halmstad[4], and was the youngest of the pastor's twelve children.

After the early death of Louise's paternal grandfather Carl Henric Pettersson (1803–1834), a mill owner, her grandmother Christina Maria Rudelius (1801–79) and her children, among whom was Edvard, moved in with her brother Carl Johan Hill (1793–1875), a professor of mathematics known as a

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Key words and phrases. Louise Petrén-Overton, biography, first Swedish female mathematician, n-order partial differential equations.

Translation and publication. The article is accepted for publication in the journal of mathematical societies of Moscow, St. Petersburg and Nizhny Novgorod "Mathematics in Higher Education", No 18, 2020. This article i a translation from Russian by professor O. V. Petrova.

[1]Barbro Grevholm — professor emerita at the Department of Mathematical Sciences, Faculty of Engineering and Science, Universitetet i Agder in Norway and of Kristianstad University in Sweden. She also runs her own company Barbro Grevholm Läromedel, which carries out expert work in didactics of mathematics and produces textbooks in mathematics.

[2]Lars Ture Jaakko Haikola was born in 1947 in Lund. Defended his thesis in religious philosophy at Lund University (1977). Was Head of the Department of Theology at Lund University, Rector of Malmö Teachers' College (1995–98), Head of Blekinge Institute of Technology in Karlskrona (2001-07). Rector of Lund University Campus Helsingborg (2007). In 2008 was appointed Head of the committee for the merger of Växjö University and the University of Kalmar to Linnaeus University in Växjö and Kalmar. In 2010–2014 — Chancellor of the Universities in Sweden and head of the Swedish National Agency for Higher Education. After retirement in 2014 made evaluation of universities and research institutions efficiency. Chairperson of the University Board at Örebro University and of Baltic Sea Action Group.

[3]Married name Hedvig Louise Beata Petrén-Overton.

[4]Halmstad is medieval parish in Sweden situated in Kågeröd in Skåne.

Louise Petrén-Overton

scientist of great erudition. During his work at Lund University (1830–1870) he was rector twice. From 1848 he was a member of the Royal Swedish Academy of Sciences.

Louise Petrén's father, Edvard, took his name Petrén after his uncle Anders Petrén (1793–1834), a teacher. Edvard studied at Lund University from 1842, was ordained in 1848 and became Master of Philosophy in 1850, after defending his doctoral thesis in mathematics entitled "On the more general use of square and the table of square numbers". Apparently, he discussed the choice of the topic of his dissertation with Professor Carl Johan Hill, in whose family he lived.

After completing his studies, Edvard Petrén served for eight years at the Lund University Library. From 1868 he was pastor of two parishes in Malmöhus län. He held several positions, including inspector of Malmöhus län state colleges and schools (1876–1896).

Louise's mother Charlotta (Kjerstin Beata Charlotta Göransson, 1838–1894) was the daughter of Jöns Göransson (1806–1864), notary public of Lund, deputy governor, and Petronella Christina Brorström (1806–1842).

There were nine sons and three daughters in the family. Louise was the youngest of the children, her sisters were much older (by 20 and 15 years). Together with their mother, they were able to cope with the household and Louise had the opportunity to study alongside her brothers. Louise felt quite comfortable; everyone in this hardworking family was brought up in an atmosphere of friendly support and respect for everyone's interests and inclinations. All the ten children who studied received a good education. This is what all her nine brothers were:

Her eldest brother *Thure* Petrén (1858–1926) was a general practitioner, the first provincial physician in Malmöhus län and a member of the Malmö City Council,

Johan *Edvard* Petrén, (1863–1930) — a lawyer, a judge in the Supreme Court,

Daniel *Alfred* Petrén, (1867–1964) — a professor of psychiatry at Uppsala University (1932–1939), a member of parliament (1912–1935),

Karl Anders Petrén, (1868-1927) — a professor of practical medicine at Uppsala University (1902–1910), a famous Swedish physician[5],

[5]Karl Anders Petrén studied medicine under such prominent physicians of the time as Magnus Blix (1849–1904) in Sweden and Joseph Jules Dejerine (1849–1917) in Paris.

Bror August Petrén, (1870–1938) — a lawyer and a politician,

Jakob Georg Petrén, (1872—1950) — a chemist, professor of rock chemistry,

Erik *Gustaf* Petrén, (1874–1962) — professor of surgery at Uppsala University (1918–21), then at Lund University (1921–39),

Esaias *Viktor* Oktavus Petrén, (1876–1960) — a lawyer, Attorney General,

Finally, the youngest brother *Ebbe* Oskar Sigfrid Petrén (1878–1974) was epidemiologist and bacteriologist, physician, assistant professor at Lund University and chief physician at Malmö Hospital (1908–43).

From early years, Louise loved mathematics — the only one in the family — and that love was encouraged by her father and brothers. One memory is passed down in the family from generation to generation. Once, when Louise was a little girl, she was seriously ill and hopes for recovery were dwindling away. One day she asked if she could take her mathematical books with her to heaven. *"If I am not allowed to bring the mathematical books with me to heaven, I do not want to go there!"* Having made this decision, she began making a quick recovery and overcame the disease. This is how Lars Haikola commented on the family legend: *"My maternal grandmother uttered these dramatic words when as a child she was down with scarlet fever. She continued to enjoy telling the story almost ninety years later... Her mind was totally clear almost to the day of her death at the age of 96"* [2].

In 1899 Louise received her General Certificate of Education as a privately tutored student at Lund Cathedral School. In 1902 she got her BA degree in the combination of four subjects: mathematics, physics, chemistry, and mechanics. At that time, Lund University had about 600 students in various fields of study; 8 to 12 of them in different years were women, and Louise was the only one who chose science. Lars Haikola wrote [2]: *"my grandmother described her university time as surprisingly uninspiring"*. Her professor of mathematics and mechanics apparently considered teaching two students, including Louise, a waste of time. In addition, he did not hide his negative attitude towards teaching a girl.

Already as a young girl Louise gained the skill to learn on her own. There was a rich science library at home. In addition, thanks to the family members, the extensive Lund University library was early made available to her. With fellow-students Louise behaved naturally, was sociable and friendly. This behavior was formed in her family, where her brothers, along with her parents, loved and cared for her. Among the highlights of her student life Louise Petrén mentioned her trip to a conference organized in Stockholm by Professor Gösta Mittag-Leffler. Louise *"found it extremely stimulating to meet other mathematicians and to discover new branches of mathematics"* [2]. Besides, in Stockholm she got a chance to attend several lectures by leading professors.

"In 1910 Louise received her Licentiate of Philosophy and in 1911 she defended her doctoral dissertation in mathematics" [2]. For many years

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Having received his doctorate in 1896, he served as a professor of medicine at Uppsala and Lund Universities. Specialist in diabetes. Ahead of his time, he recommended a low-carbohydrate and fat-rich diet to his patients. Besides, in a treatise published in 1901 he described and proposed measures for the treatment of gait disorders.

Louise Petrén-Overton as a student
https://kulturportallund.se/lundaprofil__11693/

Louise Petrén remained the only woman in Sweden with a doctorate in mathematics.

She did not have a scientific supervisor or seminars at the department in the sense we understand it today. She worked independently, her only "consultants" were books. She passed the final exam for her doctorate after defending the dissertation, in 1911.

Unfortunately, that was the end of her research career. Deep prejudices of the beginning of 20th century forever "clipped wings" of this talented scientist. Her mathematics professor categorically stated that only a man, whose duty it is to support his family, can get a place at the university. A woman can only qualify for a teaching position in a girls' school. *"At 95, my grandmother was still mortifiedover this attitude!"*, — that's how Lars Haikola commented on this episode in his grandmother's life [2].

Louise Petrén had all the prerequisites for continuing her successful scientific career: her family encouraged her studies and respected her early interest in mathematics; at the university she was given the freedom to choose her specialization. Having felt the joy of discovering new knowledge, she was planning her next steps in the chosen field of mathematics. Her work was appreciated by the professors and published, as a reward, in the annual works of Lund University ... But her hopes were not destined to come true.

What happened next? Happy marriage. Fortune smiled on Louise Petrén. In 1912 she married Charles Ernest Overton (1865–1933), a British physiologist and biologist.

Charles Ernest Overton was born in Stretton, Cheshire, UK into the family of Samuel Charlesworth Overton and Harriet Jane Fox, who was the daughter of the Reverend W. Darwin Fox. The latter was an entomologist, Charles Darwin's second cousin. Ernest Overton was educated in Wales, at Newport High School. When he was 17 years, the family moved to Switzerland. He studied biology, especially botany, at University of Zurich. After obtaining the PhD in 1889 he worked at the University as Dozent in biology. 10 years later he moved to the physiology department of Würzburg in Germany. Finally, in 1907, at the age of 42, he was invited to work at Lund University as professor and accepted a newly opened chair of pharmacology, where he worked until his retirement in 1930.

Nowadays Charles Ernest Overton is regarded a pioneer of the theory of the cell membrane. Among other things, he is the author of the hypothesis well known today as Overton's lipoid (or lipide) theory of plasma permeability. The hypothesis is that for an extremely short interval the surface of the contracting muscle fibers becomes permeable to sodium and potassium ions. *"This fundamental idea in the theory of propagation of impulses in nerves and muscles was worked out almost fifty years later by A. L. Hodgkin and A. F. Huxley, for which they were awarded the Nobel Prize in physiology or medicine in 1963"* [3]. Another Overton's research was related to the mechanism of narcosis. Almost simultaneously with Overton, but independently of him, the pharmacologist Hans Horst Meyer reached the same result. The Meyer–Overton theory became a starting point for explaining this phenomenon.

Biographers say that Ernest Overton was a gentle and placid man. Even when dealing with his opponents *"Overton never replied to the attacks on his views"*. He possessed *"a striking intuitive ability to recognize the great, fundamental problems and to envision a means of solving them without recourse to complicated apparatus. He never founded a school in the proper sense of the word, and his publications, almost all of which were written in German, do not seem to have been widely read in the original, especially in English-speaking countries. Nevertheless, his influence on the development of cell physiology and pharmacology has been strong and long-lasting. He was one of those scientists whose stature is more obvious after their death than it was during their lifetime"* [4].

Charles Ernest Overton and Louise had four children. For twenty years Louise worked as a part-time teacher of mathematics at Elisabet Lindeberg's girls' school in Lund. For some time, she also worked at a gymnasium-level school for girls in Malmö[6]. Louise was not a born teacher. It seemed to her that her pupils were too slow, could not understand explanations at the pace she was accustomed to with her older brothers. Because of her own intensive studies and research, because of her high demand on herself- she expected her pupils to be as dedicated. But she did not always manage to light in their eyes that spark of love for mathematics, which she herself possessed and which seemed to her natural for all those who learn mathematics. Lars Haikola wrote: *"She took it for granted that mathematics and intellect went hand in hand"* [2]. Nevertheless, she did seem to have some capable schoolgirls. Probably it is them that she later taught in Malmö. For some time after graduation Louise also worked at the life insurance company Thule's actuarial department (from 1912). But this work didn't give her any satisfaction either.

The entrance to the world of science was closed to her. Never in her long, almost a century-long life was she accepted "full-time" with the right to

[6]The information is taken from Barbro Grevholm's article [1]: "Hon har även arbetat vid högre allmänna läroverket för flickor i Malmö under någon period". (English translation: "For a while she also worked at a gymnasium-level school for girls in Malmö").

pursue science, to enrich her knowledge, to discuss results with colleagues, to be able to participate in conferences, to do what she dreamed of dedicating her life to. It is not by chance that in most reference books containing information about her wonderful family, with a lot of branching references, she is not mentioned as a family member worthy of equal respect with the others.

At the age of 53, Louise Petrén-Overton was widowed. Her children needed care, so in two years she quit teaching. According to Lars Haikola, who was her neighbor for almost twenty years during the end of her life, she always enjoyed the respect of her family, bordering on reverence. In 1961, when she celebrated the 50th anniversary of her doctorate, Louise Petrén was awarded the diploma “doctor Jubilaris” at Lund University. An honorary diploma in memory of Louise Petrén-Overton has now been established in Sweden. It is awarded to persons supporting women who have chosen mathematics as their field of work [1].

It was not until 1956, that is 45 years after Louise Petrén's graduation, that another Swedish woman, Sonja Lyttkens[7] received her doctor's degree in mathematics. Louise Petrén-Overton died January 14, 1977. She is buried in the eastern cementary in Lund.

Let's now turn to Louise's scientific results and have a look at them from the contemporary point of view. Her dissertation [5] is entitled[8]

Extension of Laplace's method to the equations

$$\sum_{i=0}^{n-1} A_{1i}(x,y)\frac{\partial^{i+1} z}{\partial x \partial y^i} + \sum_{i=0}^{n} A_{0i}(x,y)\frac{\partial^i z}{\partial y^i} = 0$$

(published in Lund University Yearbook in French).

Louise Petrén begins her Introduction to the dissertation by introducing the reader to Leonard Euler's result in integration method for hyperbolic linear partial differential equation of the second order

$$\frac{\partial^2 z}{\partial x \partial y} + a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} + cz = 0, \tag{1}$$

where a, b, c are functions of variables x, y. Euler proves (1769–70) that a necessary and sufficient condition of the initial equation to admit the first integral is the fulfillment of one of the two conditions for the coefficients a, b, c. These conditions are partial differential equations of the first order:

$$\begin{aligned} &\frac{\partial a}{\partial x} + ab - c = 0, \\ &\frac{\partial b}{\partial y} + ab - c = 0. \end{aligned} \tag{2}$$

Louise demonstrates the proof of this statement, and she does it concisely, laconically.

[7]Sonja Lyttkens (1919–2014) — the first woman to teach mathematics at a Swedish university (from 1963). Graduated from Uppsala University.

[8]“Archives of ALGA” published an English translation of the Introduction to the dissertation [6, p. 13–31]. It is translated from French by Nail H. Ibragimov and Gunter Leguy.

Such a beginning of the dissertation is not typical. Usually authors prefer to write something like *"as is well known, L. Euler showed ..."*, and do not present the proof of the fact, just making a reference to the original source. Such narration, with a lot of statements not proved in the text but supplemented with numerous references instead, make the readers quickly lose the thread of reasoning and begin to feel uncomfortable, unless they know the fact and remember it. Louise Petrén does not waste time giving extensive explanations; she presents the material in a concise, demonstrative way, respectfully to the reader.

In the same style, Louise Petrén introduces the reader (or reminds the more prepared ones) to the next steps taken to obtain the integrals of equation (1). She shows the essence of Pierre-Simon Laplace's "cascade method" (1773), in which the central role belongs to invariants

$$h = \frac{\partial a}{\partial x} + ab - c \qquad \text{and} \qquad k = \frac{\partial b}{\partial y} + ab - c,$$

(in equation (2) they are zero) known as "Laplace invariants". Laplace demonstrated the way to solve many hyperbolic equations by this method.

This is followed by the results of Adrien Marie Legendre (1787), Vasily Imshenetsky (1868) and other authors, ending with Jean Gaston Darboux and Édouard Jean- Baptiste Goursat. Finally Louise Petrén comes to her own results, that is she proves that the classical results described by her allow for generalization in case of partial differential equations of order $n > 2$:

$$\sum_{i=0}^{n-1} A_{1i}(x,y)\frac{\partial^{i+1} z}{\partial x \partial y^i} + \sum_{i=0}^{n} A_{0i}(x,y)\frac{\partial^i z}{\partial y^i} = 0$$

Of her predecessors in the study of special cases of equation (3), she mentions, first, Jean-Marie Le Roux (1863–1949). Le Roux mainly treated the case $A_{0i}(x,y) = 0$ (1899). Besides, Louise names Laura Pisati[9] (1905) [7] and describes her results in detail (she considers and finds particular solutions connected with those received by Le Roux; Pisati applies similarity transformations) and then proceeds to the description of her own research.

Louise Petrén's results considerably exceed those that she found in the works of her predecessors. All the five chapters of her dissertation are devoted to the discussion of facts showing that the known approaches enumerated in her literature survey are with some reservations applicable to integration of partial differential equations of higher order. The very enumeration of new results in the Introduction impresses with its abundance and the thoroughness in the description of the restrictions that have to be imposed on the problem definition. In particular, Louise puts forward the idea of generalizing the Laplace method to the case when some characteristics of n-order hyperbolic equations coincide and gives an example of successful application of such generalization.

[9]In 1905 Laura Pisati graduated from the mathematical faculty at University of Rome, taught at "Marianna Dionigi" Technical School for Girls in the capital. Member of Deutsche Mathematiker-Vereinigung and also of Circolo matematico di Palermo. She died prematurely in 1908 on the eve of the Fourth International Congress of Mathematicians in Rome, where she was supposed to speak.

Conference “Modern Group Analysis”, 2007, Karlskrona, Sweden. Head of Organizing committee Nail H. Ibragimov (left) and Vice-Chancellor Blekinge Institute of Technology Lars Haikola (Author’s photo)

From the point of view of modern methods in differential equations analysis, all the approaches known to Louise Petrén were based on guesswork, on what specialists in modern group analysis aptly defined as “gaze method”. Textbooks on differential equations are still written today *“in a cookbook style containing numerous ad hoc recipes for integrating various special types of equations by means of artificial substitutions”* [8, p. 192]. But it is not fair to judge Louise Petrén from this perspective.

Nowadays, it’s a well-established belief that

> *“Louise Petrén is an interesting person in the history of Swedish mathematics not only because she was the first Swedish woman who defended PhD in mathematics, but also because she made a profound contribution to the constructive integration theory of partial differential equations in the direction initiated be Euler and continued by Laplace, Legendre, Imschenetsky, Darboux, Goursat. In her PhD thesis she extended to higher-order equations Laplace’s method of integration of second-order linear hyperbolic equations with two independent variables. The Introduction ... is an excellent survey of Laplace’s method and its generalizations to quasi-linear partial differential equations in two independent variables. It is interesting to consider her result from the point of view of equivalence transformations and invariants of differential equations and compare with the theory of the Laplace invariants. However, L. Petren’s research was not known until recently among mathematicians working in group analysis”.*

This is how Louise Petrén’s work was assessed by professor Nail H. Ibragimov [6, p. 3]. But it happened nearly a hundred years after Louise Petrén defended her dissertation.

It is not Author surprising photo that, having such a high opinion of Louise Petrén’s dissertation, Nail H. Ibragimov took advantage of her deep and concise description of the results of her predecessors. He reproduced this description in his historical survey preceding the main material in the article “Equivalence groups and invariants of linear and non-linear equations” [9]. Nail H. Ibragimov writes: *“She ... gave a good historical exposition which I used in the present paper, in particular, concerning Euler’s priority in discovering the semi-invariants ...”* [9, p. 12].

Then Nail H. Ibragimov presents "Gallery of main figures and landmarks" [9, p. 13–15], showing how the process of finding invariants of linear hyperbolic two-order partial differential equations and generalizing equations was developing:

d'Alembert (1747) — Euler (1769) — Laplace (1773) — Laguerre (1879) — Darboux (1890) — ***Petrén (1911)*** *— Ovsyannikov (1960) — Sophus Lie (1895) — Ibragimov (1997).*

After Petrén he places the main figures, those who took a new look at the problem. A new epoch in group analysis of differential equations began. This powerful leap in the approach to integration of differential equations was unknown to Louise Petrén. With a few exceptions, the work of Sophus Lie did not affect the methods of teaching equations in the world's universities at the beginning of the 20th century. At that time the mathematics program at Lund University didn't even mention a connection of theory of differential equations with local Lie groups theory.

Between the links of the chain "Petrén" and "Ovsyannikov" there are 49 years!

This is what the description of Louise Petrén's result in this chain looks like:

> *"Petrén Louise (1880-1977). Mathematician. Made a significant contribution to the theory of invariants of partial differential equations.*
>
> *1911 — Extension of Laplace's method and Laplace's invariants to higher-order equations by Louise Petrén in her PhD thesis "Extension de la métode de Laplace"... The work contains a good historical introduction starting from Euler's work. Petrén's invariants should be investigated from group point of view"* [9, p. 14].

Why does N. H. Ibragimov place L. V. Ovsyannikov before Sophus Lie in his "Gallery of main figures and landmarks" regardless of chronological succession? The thing is that Lev Vasilyevich Ovsyannikov headed the school on practical application of Sophus Lie's works to "living" differential equations, important in applied tasks. Ibragimov characterizes Ovsyannikov in this chain in the following way:

> *"Ovsyannikov, Lev Vasilyevich (born 1919). The spearhead in the restoration of group analysis of differential equations in 1960s. Applied Lie's theory in fluid mechanics. 1960 — Ovsyannikov discovered a new invariant, q, and used the invariants p and q to the problem of group classification of hyperbolic equations."* [9, p. 15].

N. H. Ibragimov had a reason to complete the chain with his own results beginning with 1997. And by the time the article [9] was written he himself published a remarkable result: he proposed *"a solution of the Laplace problem, which consists of finding all invariants of the hyperbolic equations and constructing a basis of the invariants. Three new invariants of the first and second orders are found, and invariant-differentiation operators are constructed. It is shown that the new invariants, together with the two invariants detected by Ovsyannikov, form a basis such that any invariant of any order* *is a function of the basis invariants and their invariant derivatives."* [10, p. 11].

I managed to find references to Louise Petrén's dissertation (1911) [5] and to Laura Pisati's article (1905) [7] in the book "Algebraic Theory of Differential Equations" published by London Mathematical Society with Cambridge University Press in 2009 [11]. In the section "Factorization of Linear Systems" of the book [11] the author professor Sergey P. Tsarev refers to the papers of Louise Petrén [5] and Laura Pisati [7] (numbers [36], [35] in the list of literature for the corresponding chapter of the book) and demonstrates the development of these results in recent years.

Having discovered this and other works of S. P. Tsarev on the Internet, in which there is a link to Louise Petrén's dissertation, I wrote to him asking to share his opinion on the place of Petrén's work in today's general picture of analysis of partial differential equations of similar type. Here are excerpts from his letter:

> *"For nearly 200 years classical Laplace transformations remained unsurpassed in the possibility of explicit integration of linear two-order equations on plane. So at the end of the 20th century they seemed a completely hopeless area for further research. However lately there has been a real surge in activity connected, on the one hand, with the recently discovered possibilities of generalization of Laplace results to arbitrary order hyperbolic systems of equations on plane, and on the other hand, with new applications to solving complicated problems of mathematical physics (e.g. stochastic differential equations). As it usually happens in such cases, with the appearance of interest in this subject, "unexpectedly" well-forgotten results are discovered that could have actually opened new paths in this area (explicit integration of partial differential equations) much earlier, as well as "unexpected" links to contemporary tasks that previously seemed completely unrelated to the Laplace transformations theory.*
>
> *Dozens of papers appeared from which it became clear that the attitude to this area as dead and unpromising is far from being correct.*
>
> *Possible applications can already now be used in most currently relevant branches of mathematical physics: theory of kinetic equations describing nonlinear stochastic dynamical systems; problems of the quantum theory of scattering and wave propagation in complex multiphase media and other branches.*
>
> *Petrén's dissertation, that sort of sums up the period of the end of the 19th — the beginning of the 20th century, is certainly of great interest in this regard, because, as it turned out, the forgotten achievements of the end of the 19th — beginning of the 20th century may well serve as an impetus for the development of new methods, and the technique applied earlier may well be generalized to new cases. It is vital to reformulate old results at the modern level, both in terms of rigor (it is necessary to specify the sphere of applicability, because at that time authors often*

ignored various particular cases that are, however, important for modern applications) and in terms of constructiveness (selection of precisely defined algorithms, identification of places where solution algorithms are not complete or are not strictly valid).

Of special importance are her ideas about the possibility of generalizing Laplace's transformations theory to non-strictly hyperbolic equations of high order (when part of their characteristics coincides). It's exactly such an example that is analyzed by Petrén. It seems that it is the type of equations studied by Petrén that can be very useful when studying kinetics of some types of stochastic dynamical systems. The reduction theory of systems of integrals outlined in Petrén's dissertation and briefly presented in the last chapter (in the appendix to parabolic equations), found its continuation with backing up of proofs, precise definition of algorithms and finding of nontrivial integrable examples. To illustrate it we can refer to [12]. These and some other ideas developed or outlined in Petrén's dissertation, can be found in textbook [13]".

It should be noted that after the 2004 and 2006 publications on Louise Petrén in "Archives of ALGA" [6] and [9] other references to these works began to appear.

Let's compare this high assessment of Louise Petrén's work with what is written about it in Sweden.

Let's see what is said in Swedish Biographical Dictionary [14]. Here is the information about Louise Petrén (here is the full quote):

"Den yngsta dottern, Hedvig Louise Beata P (1880–1977) blev 1911 fil dr på avhandlingen Extension de la méthode de Laplace aux équations ... Hon var den första kvinna som disputerade inom de matematiska disciplinerna. Hennes avhandling behandlar ett klassiskt ämne inom matematiken: lösning av vissa partiella differentialekvationer genom integration. Den präglades av stor omsorg och klarhet och var helt självständig, men genom nya metoder hade hennes ansats hamnat i en återvändsgränd. Hennes "talang och arbetsförmåga hade varit värd ett bättre ämne" (Gårding). Louise P var från 1912 gift med prof. Charles Ernest Overton".

"The youngest daughter, Hedvig Louise Beata Petrén (1880–1977), became a PhD in 1911 with the dissertation "Extension of Laplace's method and auxiliary equations". She was the first woman to defend her doctoral thesis in mathematics. Her dissertation deals with a classic subject in mathematics: solving certain partial differential equations by integrating. The thesisis characterized by great reliability and clarity and was completely independent, but due to the development of new methods, her approach has come to a dead end. Her "talent and ability to work were worthy of a better subject" (Gårding). In 1912 Louise Petrén married professor Charles Ernest Overton".

The next sentence is: *"All the nine sons of Edvard Petrén reached a prominent position in society"*. From which it follows that Louise, unlike her brothers, has not reached a prominent position in society.

Here is one more assessment.

Barbro Grevholm informed the author of this article that she had previously approached the Swedish professor Lars Gårding (his name appears in the above quotation from the dictionary [14]) in 1990, inviting him to read and evaluate Louise Petrén's dissertation from a scientific point of view. At that time Lars Gårding was writing his book "Mathematics and Mathematicians. Mathematics in Sweden Before 1950 year" [15]. In this book, he wrote: *"Louise Petrén shows impressive knowledge of literature and creative talent within the discipline. Her presentation is clear and correct".* But he also makes it clear that he considers her thesis rather obsolete.

Lars Gårding favorably responded to Barbro Grevholm's request 1990 and wrote the following (see [1, p. 73]):

> *"... After Euler, Laplace introduced a variable transformation that made it easier to find common or intermediate solutions, and after Laplace, Darboux theory was improved in his dissertation in the 1840s. Others considered more general equations. Development eventually stagnated, and the theory was codified in Goursat's book in the 1890s. At that time, the question of the existence of general solutions ceased to cause interest".*

He also writes to Barbro Grevholm:

> *"The thesis of Louise Petrén is one of the end points of the old theory. She considered general and intermediate solutions of equations of a higher degree in two variables, where one derivative behaves linearly. Despite their relatively general appearance, these equations have a strong connection with the older theory, especially the Darboux theory ... It is also necessary to say that Louise Petrén demonstrates an impressive knowledge of literature and innovative abilities in the chosen field. Her thesis is also clear and well thought out. The fact that her thesis was published in the Annual works of Lund University shows that it was appreciated by the contemporary Lund ... In 1911, when the thesis was published, Torsten Brodén and, probably, Niels Erik Nørlund were professors at Lund. Hardly any of them had anything to do with this work, which was normal at the time. This system that existed at the time led to a series of excellent theses in which the authors put everything. The thesis of Louise Petrén confirms this thought ...".*

Thus, we are convinced that Louise Petrén's work [5] is quite independent, and at the time of defense it was highly appreciated. But Gårding considers the topic of the thesis as *"obsolete"*. These days, one can disagree with this. The thing is that mathematics gives many examples of results obtained by someone that are not always used immediately. The case of Louise Petrén is one of them. The above passage from Sergey P. Tzarev's letter is a vivid confirmation of this. However, Gårding considers Darboux theory to be *"old"* as well.

Nobody could foresee that finding general and particular solutions of differential equations of different structures would acquire new significance, for example, due to the widespread use of rapidly developing methods of mathematical modelling in various fields of knowledge, due to the use of approximate methods, in which deviations from exact solutions may be decisive in assessing the behavior of dynamical systems, and finding such initial general solutions of the equations would become particularly important. Theory of stability also requires the knowledge of exact "rough" approximations as a starting point of the research. If at the beginning of the 20th century there was a belief that an explicit analytical solution of equations is a usual situation, then with the development of bifurcation theory, with the study of attractors, it became obvious that this is not a general case, but a rare gift, and the study of such solutions has acquired a new impetus! And the general picture describing the behavior of dynamical systems is much more complicated. Methods of symbolic computation are being improved, with computers performing routine computational work impossible for manual calculation, not in numbers, but in analytical form and, in particular, taking upon themselves the search for general and particular solutions of differential equations that simulate applied problems.

I would like to conclude the article with the words of Louise Petrén's grandson Lars Haikola [2]: *"Her long life was unusually eventful. She had a distinct and strong character and she would appreciate to the fact that her doctoral dissertation can now reach new readers".*

I express my gratitude to Barbro Grevholm, who invited me to read Louise Petrén-Overton's dissertation and express my opinion on the place of her result in mathematics, for the materials and for her support.

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